BULK VISCOUS COSMOLOGICAL MODELS IN BARBER'S SECOND SELF CREATION THEORY

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Barber's second self creation theory with bulk viscous fluid source for an LRS Bianchi type-I metric is considered by using deceleration parameter to be constant where the metric potentials are taken as function of x and t. The coefficient of bulk viscosity is assumed to be a power function of the mass density. Some physical and geometrical features of the models are discussed.

1. Introduction

Several modification of Einstein's general relativity have been proposed and extensively studied so far by many cosmologists to unify gravitation and many other effects in the universe. Barber [1] has produced two continuous self-creation theories by modifying the Brans-Dicke theory and general relativity. The modified theories create the universe out of self-contained gravitational and matter fields. Brans-Dicke [2] theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. However, Barber [1] has included continuous creation of matter in these theories. The universe is seen to be created out of self-contained gravitational, scalar and matter fields. However, the solution of the one-body problems reveal unsatisfactory characteristics of the first theory and in particular the principle of equivalence is severely violated. The second theory retains the attractive features of the first theory and overcomes previous objections. These modified theories create the universe out of self-contained gravitational and matter fields. Recently, Brans [3] has also pointed out that Barber's first theory is in disagreement with experiment as well as inconsistent, in general, since the equivalence principle is violated. Barber's second theory is a modification of general relativity to a variable G-theory. The second is an adaptation of general relativity to include continuous

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creation and is within the observational ambit. In this theory the scalar fields do not directly gravitate, but simply divide the matter tensor, acting as a reciprocal gravitational constant. It is postulated that this scalar field couples with the trade of energy momentum tensor. In view of the consistency of Barber's second theory of gravitation, we intend to investigate some of the aspects of this theory in this paper.

Several cosmologists have studied various aspects of Robertson-Walker model in Barber's second self-creation cosmology with perfect fluid satisfying the equation of state $p = (\gamma - 1)\rho$, where $1 \leq \gamma \leq 2$. Pimentel [4] and Soleng [5] have discussed the Robertson-Walker solutions in Barber's second self-creation theory of gravitation by using power law relation between the expansion factor of the universe and the scalar field. Carvalho [6] studied a homogeneous and isotropic model of the early universe in which parameter gamma of 'gamma law' equation of state varies continuously with cosmological time and presented a unified description of early universe for inflationary period and radiation-dominating era. Singh, [7] Reddy, [8,9] and Reddy et al [10] have presented Bianchi type space-times solutions in Barber's second theory of gravitation. Reddy and Venkateswarlu [11] present Bianchi type $-VI_0$ cosmological solutions in Barber's second theory of gravitation both, in vacuum as well as in the presence of perfect fluid with pressure equal to energy density. Shanthi and Rao [12] studied Bianchi type II and III space-times in second theory of gravitation, both in vacuum as well as in presence of stiff-fluid. Recently, Shri Ram and Singh [13] have discussed spatially homogeneous and isotropic R-W model of the universe in Barber's second self-creation theory of gravitation in the presence of perfect fluid by using 'gamma-law' equation of state. Barber's second self creation theory with perfect fluid source for an LRS Bianchi type-I metric is considered recently by Pradhan and Vishwakarma [14] by using deceleration parameter to be constant where the metric potentials are taken as function of x and t.

Most studies in cosmology involve a perfect fluid. However, observed phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyse dissipative effects in cosmology. Further, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of matter and radiation during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. It is known that the introduction of bulk viscosity can avoid the big bang singularity. For further discussions, see the references in Pradhan et al.[15−18]

For simplification and description of the large scale behaviour of the actual universe, locally rotationally symmetric [henceforth referred as LRS] Bianchi I spacetime have widely studied.[19−24] When the Bianchi I spacetime expands equally in two spatial directions it is called locally rotationally symmetric. These kinds of models are interesting because Lidsey [25] showed that they are equivalent to a flat (FRW) universe with a self-interacting scalar field and a free massless scalar field, but produced no explicit example. Some explicit solu-

tions were pointed out in Refs. [26,27] Motivated by these above arguments, in this paper, we have investigated spatially flat and non-flat bulk viscous LRS Bianchi type-I cosmological models in Barber's second theory of gravitation.

2. Field Equations

We consider an LRS Bianchi type-I spacetime

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2) \tag{1}$$

where $A = A(x,t), B = B(x,t)$. The field equations in Barber's second self-creation theory [1] are

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi T_{ij}}{\phi} \tag{2}$$

and

$$\Box\phi = 4\pi\lambda T \tag{3}$$

where $\Box\phi \equiv \phi^{;k}_{;k}$ is the invariant d'Alemberian and T is the trace of the energy momentum tensor. λ is coupling constant to be determined from the experiment ($|\lambda| \leq 0.1$). In the limit $\lambda \to 0$ this theory approaches the standard general relativity theory in every respect and $G = \frac{1}{\phi}$. The stress-energy tensor in the presence of bulk stress has the form

$$T_{ij} = (\bar{p} + \rho)u_i u_j - \bar{p}g_{ij} \tag{4}$$

where

$$\bar{p} = p - \xi u^i_{;i} \quad and \quad u^i u_i \tag{5}$$

Here ρ, p, ξ and u are, respectively, the energy density, isotropic pressure, bulk viscous coefficient and four-velocity vector of the distribution. Corresponding to metric (1), the four velocity vector u_i satisfies the equation

$$g_{ij}u^i u^j = 1 \tag{6}$$

The Bianchi identities in contravariant form applied to Eq. (2) are

$$W_{;i}T^{ij} + WT^{ij}_{;i} = 0 \tag{7}$$

where $W = -8\pi\phi^{-1}$ and in general relativity $W = -8\pi G$. A comma and a semicolon denote ordinary and covariant differentiation, respectively. In a comoving coordinate system, the surviving components of the field Eqs. (2)-(7) for metric (1) are

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{B'^2}{A^2 B^2} = -8\pi\phi^{-1}\bar{p} \tag{8}$$

$$\dot{B}' - \frac{B'\dot{A}}{A} = 0 \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{B''}{A^2B} + \frac{A'B'}{A^3B} = -8\pi\phi^{-1}\bar{p} \tag{10}$$

$$\frac{2B''}{A^2B} - \frac{2A'B'}{A^3B} + \frac{B'^2}{A^2B^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = 8\pi\phi^{-1}\rho \tag{11}$$

$$\ddot{\phi} + \frac{\dot{A}\dot{\phi}}{A} + \frac{2\dot{B}\dot{\phi}}{B} + \frac{A'\phi'}{A^3} - \frac{2B'\phi'}{A^2B} - \frac{\phi''}{A} = (\rho - 3\bar{p})(\frac{8\pi\lambda}{3}) \tag{12}$$

Here and in following expressions a prime and a dot indicate partial differentiation with respect to x and t respectively.

3. Solution of the Field Equations and Discussion

In this section, we review the solutions obtained by Pradhan and Vishwakarma. [14] Eq. (9), after integration, yields

$$A = \frac{B'}{l} \tag{13}$$

where l is an arbitrary function of x

Eqs. (8) and (10), with the use of Eq. (13), reduce to

$$\frac{B}{B'}\frac{d}{dx}\left(\frac{\ddot{B}}{B}\right) + \frac{\dot{B}}{B'}\frac{d}{dt}\left(\frac{B'}{B}\right) + \frac{l^2}{B^2}\left(1 - \frac{Bl'}{B'l}\right) = 0 \tag{14}$$

If we assume $\frac{B'}{B}$ as a function of x alone, then A and B are separable in x and t, Eq. (14) gives after integration

$$B = lS(t) \tag{15}$$

where $S(t)$ is an arbitrary function of t. With the help of Eq. (15), Eq. (13) becomes

$$A = \frac{l'}{l}S \tag{16}$$

Now the metric (1) takes the form

$$ds^2 = dt^2 - S^2(t)[dX^2 + e^{2X}(dy^2 + dz^2)] \tag{17}$$

where $X = \ln l$. With the use of Eqs. (15) and (16), Eqs. (8), (11) and (12) yield

$$\frac{2\ddot{S}}{S} + \frac{\dot{S}^2}{S^2} - \frac{1}{S^2} = -8\pi\phi^{-1}\bar{p} \tag{18}$$

$$\frac{3}{S^2} - \frac{3\dot{S}^2}{S^2} = 8\pi\phi^{-1}\rho \tag{19}$$

$$\ddot{\phi} + \frac{3\dot{\phi}\dot{S}}{S} - \frac{3\phi'l}{l'S^2} - \frac{l^2}{l'S^2}\frac{d}{dx}\left(\frac{\phi'}{l'}\right) = \frac{8\pi\lambda}{3}(\rho - 3\bar{p}) \tag{20}$$

For the sake of simplicity, if we assume ϕ to be a function of t only, then Eq. (20) with the use of Eqs. (18) and (19) reduces to

$$\frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi}\dot{S}}{\phi S} - \frac{2\lambda\ddot{S}}{S} = 0 \tag{21}$$

The function $S(t)$ remains undetermined. To obtain its explicit dependence on t, one may have to introduce additional assumptions. In the following, we assume the deceleration parameter to be constant to achieve this objective i.e.

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b(\text{constant}), \tag{22}$$

where, $H = \dot{S}/S$ is the Hubble parameter. The above equation may be rewritten as

$$\frac{\ddot{S}}{S} + b\frac{\dot{S}^2}{S^2} = 0 \tag{23}$$

On integration Eq. (23) gives the exact solution

$$S(t) = \begin{cases} [a(t - t_0)]^{\frac{1}{(1+b)}} & \text{when } b \neq -1 \\ m_1 e^{m_2 t} & \text{when } b = -1 \end{cases} \tag{24}$$

where a, m_1 and m_2 are constants of integration and the constant t_0 means the freedom of choosing the time origin. Using Eq. (22) in Eqs. (18), (19) and (21) lead to

$$8\pi\phi^{-1}\bar{p} = \frac{1}{S^2} + (2b - 1)H^2 \tag{25}$$

$$8\pi\phi^{-1}\rho = 3\left(\frac{1}{S^2} - H^2\right) \tag{26}$$

$$\frac{\ddot{\phi}}{\phi} + \frac{3\dot{\phi}H}{\phi} + 2\lambda b H^2 = 0 \tag{27}$$

3.1. *Non-flat Models*

Case (i): $b \neq -1$. For singular models since $S(0) = 0$, Eq. (24) leads to

$$S = mt^{\frac{1}{(1+b)}} \tag{28}$$

Using Eq. (28) in Eqs. (27), (25) and (26) yield

$$\phi = m^{-\frac{3}{2}}t^{-\frac{3}{2(1+b)}}\left[c_1 e^{\frac{t}{2}(1+\sqrt{1-4k_1})} + c_2 e^{\frac{t}{2}(1-\sqrt{1-4k_1})}\right] \tag{29}$$

$$8\pi(p - \xi u^i_{;i}) = \phi\left[\frac{1}{m^2 t^{\frac{2}{(1+b)}}} + \frac{(2b-1)}{(1+b)^2 t^2}\right] \tag{30}$$

$$8\pi\rho = 3\phi\left[\frac{1}{m^2 t^{\frac{2}{(1+b)}}} - \frac{1}{(1+b)^2 t^2}\right] \tag{31}$$

where

$$k_1 = \frac{[2b(4\lambda + 3) - 3]}{4(1+b)^2} \tag{32}$$

and c_1 and c_2 are arbitrary constants of integration. Therefore, the geometry of the universe, in this case, is described by the line-element

$$ds^2 = dt^2 - m^2 t^{\frac{2}{(1+b)}}[dX^2 + e^{2X}(dy^2 + dz^2)] \qquad (33)$$

Here the Barber's scalar function ϕ is given by Eq. (29) and the corresponding physical parameters p and ρ are given by the Eqs. (30) and (31) respectively.

Thus, for given $\xi(t)$, we can solve the pressure p. It is standard to assume [28,29] the following ad-hoc law

$$\xi(t) = \xi_0 \rho^n \qquad (34)$$

If $n = 1$, Eq. (34) may correspond to a radiative fluid, whereas $n = \frac{3}{2}$ may correspond to a string dominated universe. [30] However, more realistic models [31] are based on n lying in the region $0 \le n \le \frac{1}{2}$.

Model I $(\xi = \xi_0)$

In this case, we assume $n = 0$. Further from Eq. (34), we obtain $\xi = \xi_O$ = constant and hence Eq.(30) becomes

$$p = \frac{\phi}{8\pi t^2}\Big[\frac{1}{m^2 t^{\frac{1}{(1+b)}}} + \frac{(2b-1)}{(1+b)^2}\Big] - \frac{2\xi_0}{(1+b)t} \qquad (35)$$

Model II $(\xi = \xi_0 \rho)$

In this case, we assume $n = 1$. Further from Eq. (34), we obtain $\xi = \xi_O \rho$ and hence Eq. (30) reduces to

$$p = \frac{\phi}{8\pi t^2}\Big[\frac{1}{m^2 t^{\frac{1}{(1+b)}}} + \frac{(2b-1)}{(1+b)^2} - \frac{9\xi}{(1+b)t}\Big\{\frac{1}{m^2 t^{\frac{1}{(1+b)}}} - \frac{1}{(1+b)^2}\Big\}\Big] \qquad (36)$$

From Eq. (31), we observe that the energy density $\rho(t)$ is decreasing function of time. As t tends to infinity, energy density will vanish. From Eq. (29) , it is observed that Barber's scalar function ϕ decreases as time increases and will vanish when t turns to infinity. The energy conditions [32,33] are satisfied provided $m^2 < 1$ and $b = 0$ with positive c_1 and c_2.

Physical behaviour of the models: In case of a non-flat model when $b \ne -1$, the Ricci scalar becomes

$$R = \frac{1}{m^2 t^{\frac{2}{(1+b)}}} - \frac{(1-b)t}{(1+b)} \qquad (37)$$

It is observed from Eq. (37) that when $t \to 0$; (i) $R \to \infty$ if $b = 0$, and (ii) $R \to \infty$ if $b \ge 1$. The Eq. (37) also suggests that when $t \to \infty$; $R \to 0$ if $b \ge 0$. The scalars of the expansion and shear are given by

$$\theta = \frac{3}{(1+b)t} , \quad \sigma = 0 \qquad (38)$$

The model has singularity at $t = 0$. At $t \to \infty$, the expansion ceases. Here, $\frac{\sigma}{\theta} = 0$, which confirms the isotropic nature of the space-time which we have already obtained in Eq. (33).

Case (ii): $b = -1$. In this case using Eq. (24) in Eqs. (27), (25) and (26) give

$$\phi = m_1^{-\frac{3}{2}} e^{-\frac{3m_2 t}{2}} \left[c_3 e^{\sqrt{k_2} t} + c_4 e^{-\sqrt{k_2} t} \right] \tag{39}$$

$$8\pi(p + 3m_2\xi) = \phi \left[\frac{1}{m_1{}^2 e^{2m_2 t}} - 3m_2{}^2 \right] \tag{40}$$

$$8\pi\rho = 3\phi \left[\frac{1}{m_1{}^2 e^{2m_2 t}} - m_2{}^2 \right] \tag{41}$$

where

$$k_2 = m_2{}^2 \left[2\lambda + \frac{9}{4} \right] \tag{42}$$

and c_3 and c_4 are arbitrary constants of integration. Therefore, the geometry of the universe, in this case, is described by the line-element

$$ds^2 = dt^2 - m_1{}^2 e^{2m_2 t} [dX^2 + e^{2X}(dy^2 + dz^2)] \tag{43}$$

In this case, the Barber's scalar function ϕ is given by the Eq. (39) and the corresponding physical parameters p and ρ are given by Eqs. (40) and (41) respectively. For $m_2 = 0$, the model reduces to a static radiating model with constant density and pressure.

Model I $(\xi = \xi_0)$

In this case, we assume $n = 0$. Further from Eq. (34), we obtain $\xi = \xi_O =$ constant and hence Eq. (40) reduces to

$$p = \frac{\phi}{8\pi} [\frac{1}{m_1^2 e^{2m_2 t}} - 3m_2^2] - 3m_2\xi_0 \tag{44}$$

Model II $(\xi = \xi_0\rho)$

In this case, we assume $n = 1$. Further from Eq. (34), we obtain $\xi = \xi_O\rho$ and hence Eq. (40) reduces to

$$p = \frac{\phi}{8\pi} [\frac{1}{m_1^2 e^{2m_2 t}} - 3m_2^2 - 9m_2\xi_0\{\frac{1}{m_1^2 e^{2m_2 t}} - m_2^2\}] \tag{45}$$

Physical behaviour of the model: The Ricci scalar R is

$$R = 2m_2{}^2 - \frac{1}{m_1{}^2 e^{2m_2 t}} \tag{46}$$

It is easily observed from Eq. (46) that (i) when $t \to 0$, $R \to (2m_2{}^2 - \frac{1}{m_1{}^2})$, and (ii) when $t \to \infty$, $R \to 2m_2{}^2$. The expansion and shear scalars are

$$\theta = 3m_2, \quad \sigma = 0 \tag{47}$$

The model represents an uniform expansion as can be seen from Eq. (47). The flow of the fluid is geodetic as the acceleration vector $f_i = (0, 0, 0, 0)$.

3.2. *Flat Models*

Motivated by the results of the BOOMERANG experiment on Cosmic Microwave Background Radiation, [34] we wish to study a spatially flat bulk viscous cosmological models.

The condition for the flat model is obtained as

$$\frac{1}{S^2} = (1 - b)H^2 \tag{48}$$

Using Eq. (48), Eqs. (25) and (26) reduce to

$$8\pi(p - \xi u^i_{;i}) = \phi b H^2 \tag{49}$$

$$8\pi\rho = -3\phi b H^2 , \quad \text{where } b < 0 \tag{50}$$

In this case the model has $p < 0$ which might describe the very early epoch of galaxy formation from the process of matter condensation.

Case (i): $b \neq -1$. Using Eq. (28) in Eqs. (49) and (50) yield

$$8\pi\left(p - \frac{3\xi}{(1+b)^2 t^2}\right) = \frac{\phi b}{(1+b)^2 t^2} \tag{51}$$

$$8\pi\rho = -\frac{3\phi b}{(1+b)^2 t^2} , \quad \text{with } b < 0 \tag{52}$$

where ϕ is already given by Eq. (29). Here, the model describes the early phase of evolution as mentioned earlier.

Model I $(\xi = \xi_0)$

In this case, we assume $n = 0$. Further from Eq. (34), we obtain $\xi = \xi_O =$ constant and hence Eq. (51) reduces to

$$p = \frac{1}{(1+b)t}\left[\frac{\phi b}{8\pi(1+b)t} + 3\xi_0\right] \tag{53}$$

Model II $(\xi = \xi_0\rho)$

In this case, we assume $n = 1$. Further from Eq. (34), we obtain $\xi = \xi_O\rho$ and hence Eq.(51) reduces to

$$p = \frac{\phi b}{8\pi(1+b)^2 t^2}\left[1 - \frac{9\xi_0}{(1+b)t}\right] \tag{54}$$

Case (ii): $b = -1$. In this case using Eq. (24) in Eqs. (49) and (50) yield

$$8\pi(p - 3m_2\xi) = -\phi m_2^2 \tag{55}$$

$$8\pi\rho = 3\phi m_2^2 \tag{56}$$

where, the Barber's scalar function ϕ is given by the Eq. (39).
Model I $(\xi = \xi_0)$

In this case, we assume $n = 0$. Further from Eq. (34), we obtain $\xi = \xi_O =$ constant and hence Eq. (55) reduces to

$$p = -\frac{\phi m_2^2}{8\pi} + 3m_2\xi_0 \tag{57}$$

Model II $(\xi = \xi_0\rho)$

In this case, we assume $n = 1$. Further from Eq. (34), we obtain $\xi = \xi_O\rho$ and hence Eq. (55) becomes

$$p = -\frac{\phi m_2^2}{8\pi}(1 - 9m_2\xi_0) \tag{58}$$

4. Conclusion

In this paper we have obtained a class of LRS Bianchi Type I models in Barber's second self creation theory in the presence of bulk viscous fluid for constant deceleration parameter. Assuming an ad-hoc law for the coefficient of bulk viscosity of the form $\xi(t) = \xi_o\rho^n$, where ρ is the energy density and n is the power index, we have obtained exact solutions of the field equations. The nature of the Barber's scalar function ϕ and energy density ρ have been examined for values of $n = 0$ and $n = 1$ for both the (i) power-law and (ii) exponential expansion of both the non-flat and flat universe. The models discussed here are isotropic and homogeneous and, in view of the assumption of isotropy, the shear viscosity is absent. The effect of bulk viscosity is to produce a change in the perfect fluid. We observe here that Murphy's conclusion about the absence of a big bang type singularity in the finite past in models with bulk viscous fluid is, in general, not true.

Acknowledgements

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